Mathematical modelling as a research field: transposition challenges and future directions

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Outline

I. Research field on Mathematical Modelling

II. Brief journey through TWG Applications and Modelling

III. Systems, models and approaches to Modelling

IV. Teaching modelling through Study and Research Paths

V. Conclusions
I. Research field on Mathematical Modelling

- In the last decades, research about Applications and modelling has grown in the research community and in curricular reforms.

- Origins and development in the International Research Community
  - Initial works of F. Klein as 1st president of ICMI, Freudenthal (1968) and Pollak (1968) in a symposium ‘Why to teach mathematics so as to be useful’.
  - Followed by the creation of the ICTMA Conferences – International Community on the Teaching of Mathematical Modelling and Applications–; and thematic working groups in ICME –TSG 22 applications and modelling in mathematics education– and in CERME –TWG Applications and Modelling.
  - Several attempts to analyse the evolution of the research field in mathematics education (Blum & Niss, 1991; Kaiser & Sriraman, 2006; Blum, 2015; Schukajlow et al. 2018).
I. Research field on Mathematical Modelling

The 14th ICMI Study. Modelling and applications in mathematics education

Cai, J. et al. (2014). PME 38, PME-NA 36

Kaiser & Schukajlow (2023) PME 45

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1. Research field on Mathematical Modelling

Blum et al. (2007). Modelling and applications in mathematics education. The 14th ICMI study.

ZDM - Mathematics Education
Modeling Research around the world
Rich transposition interaction with educational institutions

OECD / PISA [T]he notion of mathematical modelling, which has historically been a cornerstone of the PISA framework for mathematics (OECD, 2004), into the PISA 2012 definition of mathematical literacy [..] The modelling cycle is a central aspect of the PISA conception of students as active problem solvers; however, it is often not necessary to engage in every stage of the modelling cycle, especially in the context of an assessment (Blum, Galbraith & Niss, 2007, pp. 3-32).” OECD (2019, p.75-76)
Curricular reforms around the world

- **Curriculum in Europe**
  
  Modelling is included in the language of competencies (Danish KOM project, Niss 2003)

- **Outside Europe: Common Core Standards in the USA**

  “Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the [rest of] high school standards […]” CCSO, Common Core State Standards in Mathematics
Curricular reforms in Europe → Some voices from European researchers

When did mathematical modelling first become part of your curriculum?

**Portugal** (S. Carreira)
In the 60s in the experimental curriculum proposed by Sebastião and Silva. It reappears in the early 90s linked to real-life situation.

**Spain** (I. Ferrando, C. Segura)

**Sweden** (J. B. Ärlebäck)
In 2000 at the lower Secondary. In 2011 and 2022 at upper Secondary, with more explicit references to models/ling.

**Denmark** (B. Jessen)

**Germany** (G. Greefrath)
Curricular reforms in Europe ➔ Some voices from European researchers

What role has modelling played in the mathematics curricula?

**Portugal** (S. Carreira)
“It has changed over the years: as an aim of teaching mathematics, or as a transversal competence to facilitate connections. Recently, modelling appears closely linked to interdisciplinarity.”

**Spain** (I. Ferrando, C. Segura)
Modelling appears as a specific competence. It often appears linked to the domain of algebra, but also as cross-disciplinary knowledge and to connections between mathematics domains.

**Sweden** (J. B. Ärlebäck)
“There’s some tension in curricular descriptions between: problem solving, applications and modelling.”

**Denmark** (B. Jessen)
One of the 8 competencies constituting mathematics subject (Jessen & Kjeldsen, 2021).

**Germany** (G. Greefrath)
Since 2003, modelling has been described as 1 of the 6 general mathematical competencies. In 2022, a clearer description of the educational standards reform.
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II. Brief journey through TWG Applications and Modelling

From CERME 4 in 2005

- Increasing number of participants and of contributions presented.
- A total of 198 contributions, from CERME 4 to CERME 12.
- More than 25 countries represented.

To CERME 13 in 2023

CERME 13
13TH CONGRESS OF THE EUROPEAN SOCIETY FOR RESEARCH IN MATHEMATICS EDUCATION
10-14 July 2023
Budapest
Hungary

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CERME 4 → CERME 5
- Different approaches, with no homogeneous understanding of what modelling is.
- Need of classifying different approaches and research aims.

CERME 6 → CERME 7
- The contributions revealed some salient themes and theoretical approaches.

### School levels

<table>
<thead>
<tr>
<th>Level</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>Preschool</td>
<td>26%</td>
</tr>
<tr>
<td>Primary school</td>
<td>7%</td>
</tr>
<tr>
<td>Inservice teacher education</td>
<td>9%</td>
</tr>
<tr>
<td>University</td>
<td>56%</td>
</tr>
<tr>
<td>Secondary school</td>
<td>2%</td>
</tr>
</tbody>
</table>

### Use of theoretical frameworks

<table>
<thead>
<tr>
<th>Framework</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>MatMod cycle &amp; competences</td>
<td>15%</td>
</tr>
<tr>
<td>ATD</td>
<td>20%</td>
</tr>
<tr>
<td>RME</td>
<td>12%</td>
</tr>
<tr>
<td>CHAT</td>
<td>3%</td>
</tr>
<tr>
<td>MMP</td>
<td>4%</td>
</tr>
<tr>
<td>Others / Not clear</td>
<td>19%</td>
</tr>
</tbody>
</table>
Diversity of approaches remains a feature of the group.

Three salient themes:
- Mathematical modelling related to problem-solving, project-based and inquiry-based approaches.
- Authenticity of modelling tasks and tools for the design and analysis.
- Theory and practice on modelling for teacher education and mathematics educators.
Period 3

CERME11 $\rightarrow$ CERME12 CERME13

- **Relevance of modelling and applications** after important societal phenomena
  - Data science, forecasting and simulations
  - Scientific community working on modelling for decision-making (such as COVID pandemic)
- Increase of research focus on **Teacher Education for mathematical modelling**
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Possible divergences?

- Diversity of theoretical approaches with different understandings about what mathematical modelling is.
- Variety of research questions and methodological tools grounded on the theoretical approaches.
- Different units of analysis are considered: empirical domains not easily comparable.
III. Systems, models and approaches to modelling

**Systems, models and their dialectic** (Chevallard, 1989)

- **A simple and flexible perspective**
  - Two main elements: that of the system and of the model
  - The notions of system and model are functions, not qualities or entities

- **A unifying and general perspective**
  - Modelling in mathematics as in any scientific activity (including Didactics)

How do different research frameworks on modelling interpret the relation between systems and models?
III. Systems, models and approaches to modelling

- The nature of Systems
  - A system is considered as a piece of reality, which can be separated from everything else (hypothetically).

- The function of Models
  - The interest and richness of a model lies in its capacity to produce knowledge about the system being modelled.
  - The problems of adapting, contrasting, validating models against the system are the driving forces of the modelling activity.
III. Systems, models and approaches to modelling

Two central properties of the relation system-model

- **Recursivity of the modelling process.** Working on the model can lead to the construction of successive models, better adapted to the system.

System | M1 | M2 | M3 | M4 | 58% and 42%
---|---|---|---|---|---
15/26 and 11/26
Two central properties of the relation system-model

- **Recursivity of the modelling process.** Working on the model can lead to the construction of successive models, better adapted to the system.

- **Reversibility of the modelling relationship.** The link between system(s) and model(s) can be inverted. The system can appear as a model of its model.
Two central properties of the relation system-model

- **Recursivity of the modelling process.** Working on the model can lead to the construction of successive models, better adapted to the system.

- **Reversibility of the modelling relationship.** The link between system(s) and model(s) can be inverted. The system can appear as a model of its model.
Research on mathematical modelling provides evidence of strong institutional constraints hindering its broad and long-term dissemination.

We know how to teach modelling, have shown how to develop the support necessary to enable typical teachers to handle it […] The bad news? ‘Many’ is compared with one; the proportion of classrooms where modelling happens is close to zero. (Burkhardt, 2008, CERME6)

The difficulties of implementing widely-agreed changes [such as modelling] seems to be a property of school systems and the way ‘this kind of organism’ functions. (Ibid, 2018, p.74)
III. Systems, models and approaches to modelling

- **Tensions** in discourses about modelling (Galleguillos & Borba, 2018)
- **Barriers and Levers** (Burkhardt, 2008)
- **Counter-arguments of students** (Blum, 1991)
  - **Teacher’s beliefs obstacles** (Kaiser & Maaß, 2007)
  - **Teachers’ dilemmas** (Blomhøj & Kjeldsen, 2006)

Detection of constraints, but they are neither structured as research questions, nor approached with specific analysis tools
III. Systems, models and approaches to modelling

Scholarly knowledge
- Institutions producing & using M

Knowledge to be taught
- Educational system, “Noosphere”

Taught knowledge
- Classroom, teaching

Learn, available knowledge
- Community of study

Chevallard (1985), Chevallard & Bosch (2020)

Tensions in discourses about modelling
- (Galleguillos & Borba, 2018)

Barriers and Levers
- (Burkhardt, 2008)

Counter-arguments of students
- (Blum, 1991)

Teacher’s beliefs obstacles
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→ Detection of constraints, but they are neither structured as research questions, nor approached with specific analysis tools
III. Systems, models and approaches to modelling

How to make explicit the implicit? Barquero, Bosch & Gascón (2019), Barquero & Jessen (2019)

Epistemological dimension

- What are the epistemological conceptions of mathematical modelling?
- How to interpret the relation between systems and models?

Ecological dimension

- What are the conditions and constraints discussed? What agents and institutions are observed/analysed?
III. a. Advances and contributions from the Mod cycle approach


Blum (1985), Kaiser-Meßmer (1986)

Maaß (2006)

Blum & Leiß (2005), Borromeo Ferri (2006)
Case 1: Modelling cycle approach → Epistemological dimension

- Modelling cycle(s) seems particularly useful to analyse cognitive processes followed by students and teachers.

Research questions: How do students solve modelling tasks? What are the influences of the mathematical thinking style of students on the modelling process? Which are the individual modelling routes? […]
Case 1: Modelling cycle approach

Context: 10th grade, 3 modelling tasks, 90 min class sessions (Borromeo Ferri, 2010)

Lighthouse
In the bay of Bremen, directly on the coast, a lighthouse […]. How far was a ship from the coast when it saw the lighthouse for the first time? (in Borromeo Ferri, 2010, p. 109-110)

Bales of straw
Straw bails are piled up that in the bottom line are five, in the next four, then three, then two and on the top one ball. Try to find out exactly, how high this mountain of straw bales is. (task in Borromeo Ferri, 2007, p. 2084)
III. a. Advances and contributions from the Mod cycle approach

**Case 1: Modelling cycle approach extended with technology**

- The use and role of technology in modelling processes and the extension of the modelling cycle to include the ‘technology world’.

![Extended modelling cycle - regarding technology when modelling](CERME6)

Possible use of digital tools in modelling cycles (Greefrath, 2011, p. 303)
Ill. a. Advances and contributions from the Mod cycle approach

Case 1: Modelling cycle approach → Ecological dimension

- The **unit of analysis** chosen focuses on student and/or teachers learning and teaching modelling processes.

- The **conditions** and **constraints** are coherently discussed about individuals (students or teachers) reacting in classroom settings or training contexts.
  - **Kinds of blockages** of students advancing in modelling (Galbraith & Stillman, 2006)
  - Mathematical **teachers’ or students’ beliefs** when modelling (Kaiser & Maaß, 2007)
  - **Teachers’ beliefs** on the use of technology (in modelling) (Siller & Greefrath, 2010)
Case 2: Models and modelling perspective → Epistemological dimension

- In the Models and Modelling Perspective (MMP), mathematical modelling is described as a sequence of model developments, involving different types of activities: model creation, model exploration and model adaptation (Lesh et al., 2003).

Connecting, coordinating and integrating models

(Ärlebäck & Doerr, 2015, CERME 9)

[T]he different transitions should not be thought of as carried out sequentially, but rather as processes fundamentally evolving simultaneously, nested and organically (Ärlebäck & Doerr, 2015)

General structure of a model development sequence (Doerr & English, 2003)
Case 2: Models and modelling perspective


- **Research Question:** How do students interpret average rates of change related to decreasing functions? How can the design of a model development sequence about negative rates of change?

- **Context:** A six-weeks entrance course on mathematics at university.
Case 2: Models and modelling perspective → Ecological dimension

- Need to create alternative epistemological models to describe the knowledge to be taught → covariational framework (Carlson et al., 2002)

- Detection of important didactic phenomena and associated constraints:
  - Lack of terminology to refer to variation and co-variation.
  - Students’ difficulties in communicating about the context of changing phenomena.
III.c. Advances and contributions from the ATD

Case 3: Modelling in the Anthropological Theory of the Didactic (ATD)

- Main steps in the modelling process (Chevallard, 1989)

  INITIAL QUESTION

  - Delimitation of the **system to be studied**, specifying its relevant aspects.

    Questions about the system

  - Model construction, work **on** **within** the model

    Questions about the model

  - Work with the model to produce knowledge about the system

    Questions about the systems and models

FINAL ANSWERS and new questions
Case 3: Modelling in the Anthropological Theory of the Didactic

- In the ATD, modelling is reformulated as a process of constructing and articulating mathematical praxeologies in order to answer some initial questions (Barquero, 2009; García et al., 2006).

Initial system $S_0$

$Q_0$ How many leaves do we need to feed the silkworms?

Model 1 $M_1$ $(t, s(t)) \rightarrow L(s(t))$

$M_1$ Counting and tabular models for $(t, s(t)) \rightarrow L(s(t))$

$Q_1$ If we have cocoons, how does it change the situation?
III.c. Advances and contributions from the ATD

System 1 $S_1$

M1 Counting and tabular models for $(t, s(t)) \rightarrow L(s(t))$

Q₁ If we have cocoons, how does it change the collection count?

Model 2 $M_2$ $(t, s(t), c(t)) \rightarrow L(s(t))$

M2 Counting sub-collections and tabular models $(t, s(t), c(t)) \rightarrow L(s(t))$

Q₂ And, if we start having butterflies?
III.c. Advances and contributions from the ATD

Q_2 What happens when we start having butterflies?

Recursivity in the modelling process

System S_0

System 1 S_1

System 2 S_2

M_1 Counting and tabular models for (t, s(t)) \rightarrow L(s(t))

M_2 Counting and tabular models for (t, s(t), c(t)) \rightarrow L(s(t))

Model 3 M_3

(t, s(t), c(t), b(t))

New butterflies

Still alive butterflies

Deceased butterflies

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III.c. Advances and contributions from the ATD

Reversibility

Model 1 \( M_1 \)
\((t, s(t)) \rightarrow L(s(t))\)

Model 2 \( M_2 \)
\((t, s(t), c(t)) \rightarrow L(s(t))\)

Model 3 \( M_3 \)
\((t, s(t), c(t), b(t))\) with
\[ b(t) = b_{\text{new}}(t) + b_{\text{alive}}(t) + b_{\text{dead}}(t) \]
The ecological dimension at the core of the ATD

Scholarly knowledge
Institutions producing, using

Knowledge to be taught
Educational system, “Noosphere”

Taught knowledge
Classroom teaching

Learnt knowledge
Community of study

Prevalence of theoretical construction of knowledge

Curriculum as a set of conceptual organisations

Rigidity of the conceptual organisation of mathematical knowledge

“Applicationism” as dominant epistemological models

Isolation of modelling

Fake inquiries, fake models and modelling as a means to study concepts

Where do they come from?

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The paradigm of **visiting works**

Programmes as a set of answers

Programmes = \{ \text{Questions}, \text{Pre-established answers} \}

A secondary role of mathematical modelling

The paradigm of **questioning the world**

Programmes as a set of questions and answers

Programmes = \{ \text{Questions, Answers to Q} \}

Primordial role of mathematics as a modelling tools to inquire into Q
IV. Teaching modelling through Study and Research Paths

Research hypothesis: Study and Research Paths (SRP) are proposed as teaching devices (Chevallard, 2005, 2006, 2015) to:

- Address didactic phenomena linked to the prevailing paradigm of visiting works
- Create the conditions facilitating the implementation and short- and long-term dissemination of mathematical modelling practices
### IV. Teaching modelling through Study and Research Paths

<table>
<thead>
<tr>
<th>SRP&lt;sub&gt;1&lt;/sub&gt;</th>
<th>Something more than silkwarms</th>
<th>Preschool (4-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td>Garcia &amp; Ruiz-Higueras (2010)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SRP&lt;sub&gt;2&lt;/sub&gt;</th>
<th>The cake box</th>
<th>Primary school (9-12)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image2.png" alt="Image" /></td>
<td>with Bosch &amp; Wozniak (2019, in press)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SRP&lt;sub&gt;3&lt;/sub&gt;</th>
<th>What padlock is safer?</th>
<th>Secondary school (15-16)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td>Vásquez et al. (2021)</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>SRP&lt;sub&gt;4&lt;/sub&gt;</th>
<th>T-shirt sizing</th>
<th>Primary-Secondary school</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4.png" alt="Image" /></td>
<td>Barquero &amp; Bosch (2023)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SRP&lt;sub&gt;5&lt;/sub&gt;</th>
<th>Facebook users</th>
<th>Secondary school-University</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5.png" alt="Image" /></td>
<td>with Ruiz-Munzón &amp; Serrano (2021)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SRP&lt;sub&gt;6&lt;/sub&gt;</th>
<th>Dynamic populations</th>
<th>University - Maths courses</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image6.png" alt="Image" /></td>
<td>Barquero (2009)</td>
<td></td>
</tr>
</tbody>
</table>
Princeton researchers predict Facebook will have 80% less users by 2017
IV. Teaching modelling through Study and Research Paths

Barquero, Monreal, Ruiz-Munzón, & Serrano (CERME10-2017, 2018)

Generating question of the SRP

Can the forecasts published by Princeton in 2014 about the future evolution of Facebook users be true? How can we model and fit real data from Facebook users to provide our forecasts and check them against Princeton’s ones?

Conditions for the implementation

- With 1st year students of Business Administration (UPF). Mathematics, from 2015/16 to 2017/18. Modelling workshops as complement to the course.
- With in-service teachers of Secondary and University, in the module of experiencing an SRP. From 2018/19 to 2022/23.
IV. Teaching modelling through Study and Research Paths

Q0 What to select and organise the data?

Q1 What data on Facebook users can we use? What hypothesis and models can we use to predict how users will evolve?

Q1.1 What to select and organise the data?

A1.1 Data-driven decisions-making

Q1.1.1 What variables to be considered?

Q1.1.2 Any specific hypotheses to investigate?

Q1.2 What has been the historical trend in the data? What other answers exist?

A1.2 Search for fitting / forecasting models → Models

Q2 What models, based on what assumptions?
IV. Teaching modelling through Study and Research Paths

**Q₂** Which mathematical models fit the data?

**Q₂.1** What family of functions does it make sense to consider? What are the ‘best’ functions to choose?

**Q₂.2** How can the coefficients of these models be determined and interpreted in this context?

**A₂** Decision-making on the ‘best’ models and interpreting the coefficients → **Models₂ and Q₃**
IV. Teaching modelling through Study and Research Paths

Q0

Q1

Q2

Q3

Q3.1

Q3.2

Q4

Q3 What does it mean to have a ‘good’ model? How do we choose the ‘best’ model?

A3 Models to fit | Models to forecast →

Models2

Q3.1 How to calculate and interpret the fitting errors made when comparing simulations with data?

Q3.2 How to use models about data variation to make decisions? (Serrano, 2010, CERME6) → Models3

Q4 How to validate short- and long-term model forecasting?
IV. Teaching modelling through Study and Research Paths

The question is still open… (to be continued)
IV. Teaching modelling through Study and Research Paths

Paradigm of questioning the world → Infinite recursivity

Questions-Answers maps
(Winsløw et al., 2013; Florensa et al., 2021)
IV. Teaching modelling through Study and Research Paths

Paradigm of questioning the world → Emergence of constraints

Many constraints for modelling at different levels of the scale...

‘Applicationism’ as prevailing epistemological model

Disciplinary confinement

Strong changes on the didactic contract

Didactic system $Q_i \rightarrow A$

Lack of routines and devices to make some gestures possible

Lack of terminology for students and teachers about modelling

Rigidity of the conceptual organisation of mathematical knowledge

Levels of didactic codeterminacy

(Chevallard, 2002)
V. Conclusions: Parallel and complementary worlds

What is mathematical modelling and how is it conceptualised?
... as an object to be taught and learnt or as a means to teach mathematics
... from research, from curricula, from the schools’ context

Collective construction of an epistemological understanding

How to diffuse mathematical modelling as a normalised activity?
... at Preschool, Primary, Secondary school and University
... in Teacher Education: pre-service or in-service teachers

Collective construction of educational infrastructures and identification of conditions/constraints

Epistemological needs

Systems ↔ Models

Recursivity and Reversibility

Didactic needs

Knowledge to be taught
Educational system, "Noosphere"

Scholarly knowledge
Institutions producing, using

Educational system, "Noosphere"

Taught knowledge
Classroom, teaching

Learn, available knowledge
Community of study

Ecological needs

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Thank you very much
Nagyon szépen köszönöm

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