

Mathematical modelling as a research field: transposition challenges and future directions

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- I. Research field on Mathematical Modelling
- II. Brief journey through TWG Applications and Modelling
- III. Systems, models and approaches to Modelling
- IV. Teaching modelling through Study and Research Paths
- V. Conclusions





- In the last decades, research about Applications and modelling has grown in the research community and in curricular reforms.
- Origins and development in the International Research Community
  - Initial works of F. Klein as 1<sup>st</sup> president of ICMI, Freudenthal (1968) and Pollak (1968) in a symposium 'Why to teach mathematics so as to be useful'.
  - Followed by the creation of the ICTMA Conferences International Community on the Teaching of Mathematical Modelling and Applications–; and thematic working groups in ICME –TSG 22 applications and modelling in mathematics education– and in CERME –TWG Applications and Modelling.
  - Several attempts to analyse the evolution of the research field in mathematics education (Blum & Niss, 1991; Kaiser & Sriraman, 2006; Blum, 2015; Schukajlow et al. 2018).











#### Rich transposition interaction with educational institutions

OECD / PISA [T]he notion of mathematical modelling, which has historically been a cornerstone of the PISA framework for mathematics (OECD, 2004), into the PISA 2012 definition of mathematical literacy [..] The modelling cycle is a central aspect of the PISA conception of students as active problem solvers; however, it is often not necessary to engage in every stage of the modelling cycle, especially in the context of an assessment (Blum, Galbraith & Niss, 2007, pp. 3-32)." OECD (2019, p.75-76)



The mathematization cycle (OECD, 2003, p. 38)





#### **Curricular reforms around the world**

Curriculum in Europe

Modelling is included in the language of competencies (Danish KOM project, Niss 2003)



## Outside Europe: Common Core Standards in the USA

"Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the [rest of] high school standards [...]" CCSO, Common Core State Standards in Mathematics



Curricular reforms in Europe → Some voices from European researchers

When did mathematical modelling first become part of your curriculum?

Portugal (S. Carreira) In the 60s in the experimental curriculum proposed by Sebastião and Silva. It reappears in the early 90s linked to real-life situation.

Spain (I. Ferrando, C. Segura) In 2007 and 2014, isolated and partial references to modelling. Explicit role in the latest curriculum reform in 2020.

Sweden (J. B. Ärlebäck) In 2000 at the lower Secondary. In 2011 and 2022 at upper Secondary, with more explicit references to models/ling.

> Denmark (B. Jessen) In 1976 at Primary and lower Secondary. In 1984-87 at Upper Secondary. Earlies 2000s, KOM framework.

Germany (G. Greefrath) In 2003 at lower Secondary. In 2012 at upper Secondary education.



#### Curricular reforms in Europe → Some voices from European researchers

What role has modelling played in the mathematics curricula?

Portugal (S. Carreira) "It has changed over the years: as an aim of teaching mathematics, or as a transversal competence to facilitate connections. Recently, modelling appears closely linked to interdisciplinarity".

Spain (I. Ferrando, C. Segura) Modelling appears as a specific competence. It often appears linked to the domain of algebra, but also as cross-disciplinary knowledge and to connections between mathematics domains. Sweden (J. B. Ärlebäck) "Some tension in curricular descriptions between: problem solving, applications and modelling".

> Denmark (B. Jessen) One of the 8 competencies constituting mathematics subject (Jessen & Kjeldsen, 2021).

Germany (G. Greefrath) Since 2003, modelling has been described as 1 of the 6 general mathematical competencies. In 2022, a clearer description of the educational standards reform.







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## II. Brief journey through TWG Applications and Modelling

#### From CERME 4 in 2005

#### To CERME 13 in 2023

TH CONGRESS OF THE EUROPEAN SOCIETY

CERME 13

CERME 4

European Research in Mathematics Education IV

Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education

Sant Feliu de Guíxols, Spain - 17 - 21 February 2005

- Increasing number of participants and of contributions presented.
- A total of **198 contributions**, from CERME 4 to CERME 12.
- More than 25 countries represented.





10-14 July 2023

**Budapest** 

Hungary



#### $\mathsf{CERME} 4 \rightarrow \mathsf{CERME} 5$

- Different approaches, with no homogeneous understanding of what modelling is.
- Need of classifying different approaches and research aims.

### CERME $6 \rightarrow$ CERME 7

 The contributions revealed some salient themes and theoretical approaches.





## $\mathsf{CERME} \overset{\mathsf{8}}{\rightarrow} \mathsf{CERME} \overset{\mathsf{10}}{\phantom{\mathsf{10}}}$

- **Diversity of approaches** remains a feature of the group.
- Three salient themes:
  - Mathematical modelling related to problem-solving, project-based and inquiry-based approaches.
  - Authenticity of modelling tasks and tools for the design and analysis.
  - Theory and practice on modelling for teacher education and mathematics educators.







### CERME11 → CERME12 CERME13

- Relevance of modelling and applications after important societal phenomena
  - Data science, forecasting and simulations
  - Scientific community working on modelling for decision-making (such as COVID pandemic)
- Increase of research focus on Teacher Education for mathematical modelling





## **II.** Brief journey through TWG Applications and Modelling

## III. Systems, models and approaches to Modelling

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- Diversity of theoretical approaches with different understandings about what mathematical modelling is.
- Variety of research questions and methodological tools grounded on the theoretical approaches.
- Different units of analysis are considered: empirical domains not easily comparable.



III. Systems, models and approaches to modelling

## Systems, models and their dialectic (Chevallard, 1989)

- A simple and flexible perspective
  - Two main elements: that of the system and of the model
  - The notions of system and model are functions, not qualities or entities

- A unifying and general perspective
  - Modelling in mathematics as in any scientific activity (including Didactics)

How do different research frameworks on modelling interpret the relation between systems and models?



## III. Systems, models and approaches to modelling



- The nature of Systems
  - A system is considered as a piece of reality, which can be separated from everything else (hypothetically).

- The function of Models
  - The interest and richness of a model lies in its capacity to produce knowledge about the system being modelled.
  - The problems of adapting, contrasting, validating models
     against the system are the driving forces of the modelling activity.





## Two central properties of the relation system-model

Recursivity of the modelling process. Working on the model can lead to the construction of successive models, better adapted to the system.





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- Reversibility of the modelling relationship. The link between system(s) and model(s) can be inverted. The system can appear as a model of its model.







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 Research on mathematical modelling provides evidence of strong institutional constraints hindering its broad and long-term dissemination

We know how to teach modelling, have shown how to develop the support necessary to enable typical teachers to handle it [...] The bad news? 'Many' is compared with one; the proportion of classrooms where modelling happens is close to zero. (Burkhardt, 2008, **CERME6**) The difficulties of implementing widely-agreed changes [such as modelling] seems to be a property of school systems and the way 'this kind of organism' functions. (Ibid, 2018, p.74)









**Tensions** in discourses about modelling (Galleguillos & Borba, 2018)

Barriers and Levers (Burkhardt, 2008)

→ Detection of constraints, but they are neither structured as research questions, nor approached with specific analysis tools



**Counter-arguments of students** (Blum, 1991)

Teacher's beliefs obstacles (Kaiser & Maaß, 2007)

**Teachers' dilemmas** (Blomhøj & Kjeldsen, 2006)













How to make explicit the implicit? Barquero, Bosch & Gascón (2019), Barquero & Jessen (2019)

#### **Epistemological dimension**

What are the epistemological conceptions of mathematical modelling?
 How to interpret the relation between systems and models?

## **Ecological dimension**

What are the conditions and constraints discussed? What agents and institutions are observed/analysed?



## III. a. Advances and contributions from the Mod cycle approach

Greefrath & Vorhölter (2016). Teaching and Learning Mathematical Modelling. ICME-13 Surveys.





## III. a. Advances and contributions from the Mod cycle approach

## Case 1: Modelling cycle approach → Epistemological dimension

 Modelling cycle(s) seems particularly useful to analyse cognitive processes followed by students and teachers.



Modelling cycle to analyse modelling routes (Borromeo Ferri, **2007-CERME5**, 2010)

 Research questions: How do <u>students</u> solve modelling tasks? What are the influences of the <u>mathematical thinking style</u> of students on the modelling process? Which are the <u>individual modelling routes</u>? [...]



#### **Case 1: Modelling cycle approach**

Context: 10<sup>th</sup> grade, 3 modelling tasks, 90 min class sessions (Borromeo Ferri, 2010)



#### Lighthouse

In the bay of Bremen, directly on the coast, a lighthouse [...]. How far was a ship from the coast when it saw the lighthouse for the first time? (in Borromeo Ferri, 2010, p. 109-110)





#### **Bales of straw**

Straw bails are piled up that in the bottom line are five, in the next four, then three, then two and on the top one ball. Try to find out exactly, how high this mountain of straw bales is. (task in Borromeo Ferri, 2007, p. 2084)





#### **Case 1:** Modelling cycle approach extended with technology

The use and role of technology in modelling processes and the extension of the modelling cycle to include the 'technology world'.





III. a. Advances and contributions from the Mod cycle approach

## Case 1: Modelling cycle approach → Ecological dimension

 The unit of analysis chosen focuses on student and/or teachers learning and teaching modelling processes.



- The conditions and constraints are coherently discussed about individuals (students or teachers) reacting in classroom settings or training contexts.
  - Kinds of blockages of students advancing in modelling (Galbraith & Stillman, 2006)
  - Mathematical teachers' or students' beliefs when modelling (Kaiser & Maaß, 2007)
  - **Teachers' beliefs** on the use of technology (in modelling) (Siller & Greefrath, 2010)



#### **Case 2: Models and modelling perspective >** Epistemological dimension

 In the Models and Modelling Perspective (MMP), mathematical modelling is described as a sequence of model developments, involving different types of activities: model creation, model exploration and model adaptation (Lesh et al., 2003).



General structure of a model development sequence (Doerr & English, 2003) Connecting, coordinating and integrating models (Ärlebäck & Doerr, 2015, CERME 9)



[T]he different transitions should not be thought of as carried out sequentially, but rather as processes fundamentally evolving simultaneously, nested and organically (Ärlebäck & Doerr, 2015)



III. b. Advances and contributions from the MMP

#### **Case 2:** Models and modelling perspective

Ärlebäck, Doerr & O'Neil (2013), Ärlebäck & Doerr (2015 and 2018)

- Research Question: How do students interpret average rates of change related to decreasing functions? How can the design of a model development sequence about negative rates of change?
- **Context:** A six-weeks entrance course on mathematics at university.





## III. b. Advances and contributions from the MMP

### Case 2: Models and modelling perspective → Ecological dimension

 Need to create alternative epistemological models to describe the knowledge to be taught → covariational framework (Carlson et al., 2002)



- Detection of important didactic phenomena and associated constraints:
  - Lack of terminology to refer to variation and co-variation.
  - Students' difficulties in communicating about the context of changing phenomena.

#### **Case 3:** Modelling in the Anthropological Theory of the Didactic (ATD)

Main steps in the modelling process (Chevallard, 1989)

#### INITIAL QUESTION

- Delimitation of the system to be studied, specifying its relevant aspects.
  Questions about the system
- Model construction, work on(within the model

Questions about the model

• Work with the model to produce knowledge about the system

Questions about the systems and models

#### FINAL ANSWERS and new questions



#### **Case 3:** Modelling in the Anthropological Theory of the Didactic

In the ATD, modelling is reformulated as a process of constructing and articulating mathematical praxeologies in order to answer some initial questions (Barquero, 2009; García et al., 2006).





#### García & Ruiz-Higueras (2010, CERME 6). More than silkworms























## The ecological dimension at the core of the ATD



**Curriculum** as a set **Rigidity of the conceptual** theoretical of conceptual organisation of mathematical **construction** of organisations knowledge knowledge "Applicationism" as

dominant epistemological models Isolation of modelling

Fake inquiries, fake

models and modelling

as a means to study

concepts

## Where do they come from?

CERME 13



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Primordial role of mathematics as a modelling tools to inquire into Q





**Research hypothesis: Study and Research Paths** (SRP) are proposed as teaching devices (Chevallard, 2005, 2006, 2015) to:

- Address didactic phenomena linked to the prevailing paradigm of visiting works
- Create the conditions facilitating the implementation and short- and longterm dissemination of mathematical modelling practices







# Princeton researchers predict Facebook will have 80% less users by 2017





Barquero, Monreal, Ruiz-Munzón, & Serrano (CERME10-2017, 2018)

## Generating question of the SRP

Can the forecasts published by Princeton in 2014 about the future evolution of Facebook users be true? How can we model and fit real data from Facebook users to provide our forecasts and check them against Princeton's ones?

## Conditions for the implementation

- With 1<sup>st</sup> year students of Business Administration (UPF). Mathematics, from 2015/16 to 2017/18. Modelling workshops as complement to the course.
- With in-service teachers of Secondary and University, in the module of experiencing an SRP. From 2018/19 to 2022/23.











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 $\mathbf{Q}_{1.1}$  What to select and organise the data?

**A**<sub>1.1</sub> Data-driven decisions-making

 $\mathbf{Q}_{1.1.1}$  What variables to be considered?

 $\mathbf{Q}_{1.1.2}$  Any specific hypotheses to investigate?

- **Q**<sub>1.2</sub> What has been the historical trend in the data? What other answers exist?
- A<sub>1.2</sub> Search for fitting / forecasting models → Models<sub>1</sub>

**Q**<sub>2</sub> What models, based on what assumptions?





- **Q**<sub>2</sub> Which mathematical models fit the data?
  - $\mathbf{Q}_{2,1}$  What family of functions does it make sense to consider? What are the 'best' functions to choose?
  - $\mathbf{Q}_{2,2}$  How can the coefficients of these models be determined and interpreted in this context?

A<sub>2</sub> Decision-making on the 'best' models and interpreting the coefficients  $\rightarrow$  Models<sub>2</sub> and Q<sub>3</sub>





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		Parameters that define the funtion		
		а	1,9	
		b	-6	
		с	1100	
REAL DATA CUBIC				
x (terms)	y (real data)	y (model)	Absolute_error	Quadratic_error
0	800	689,6	110,4	12188,16
1	845	862,5	17,5	306,25
2	901	978,4	77,4	5990,76
3	955	1048,7	93,7	8779,69
4	1007	1084,8	77,8	6052,84
5	1056	1098,1	42,1	1772,41
6	1110	1100	10	100
7	1155	1101,9	53,1	2819,61
8	1189	1115,2	73,8	5446,44
9	1228	1151,3	76,7	5882,89
10	1276	1221,6	54,4	2959,36
10	1317	1221,6	95,4	9101,16
			65,2	5116,6





#### Uso de modelos para hacer previsiones sobre los usuarios de Facebook



- Q<sub>3</sub> What does it mean to have a 'good' model? How do we choose the 'best' model?
- A<sub>3</sub> Models to fit | Models to forecast → Models<sub>2'</sub>
  - **Q**<sub>3.1</sub> How to calculate and interpret the fitting errors made when comparing simulations with data?
  - Q<sub>3.2</sub> How to use models about data variation to make decisions? (Serrano, 2010, CERME6) → Models<sub>3</sub>
  - **Q**<sub>4</sub> How to validate short- and long-term model forecasting?







## TECHNOLOGY The Washington Post Facebook loses users for the first time in its history

Facebook parent Meta's stock suffered its biggest single-day loss yet as the company refocuses on the 'metaverse'

Feb 2022

By Elizabeth Dwoskin, Will Oremus and Rachel Lerman

Updated February 3, 2022 at 4:44 p.m. EST | Published February 2, 2022 at 4:35 p.m. EST



#### New forecasts from 2023-2026

The question is still open... (to be continued)



#### Paradigm of questioning the world $\rightarrow$ Infinite recursivity



## Paradigm of questioning the world $\rightarrow$ Emergence of constraints





## V. Conclusions: Parallel and complementary worlds

## What is mathematical modelling

#### and how is it conceptualised?

... as an object to be taught and learnt or as a means to teach

mathematics

... from research, from curricula, from the schools' context

Collective construction of an epistemological understanding

#### **Epistemological needs**



**Recursivity and Reversibility** 

#### How to diffuse mathematical modelling

#### as a normalised activity?

... at Preschool, Primary, Secondary school and

University

... in Teacher Education: pre-service or in-

#### service teachers

Collective construction of educational infrastructures and identification of conditions/constraints

#### Didactic needs Ecological needs







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# Thank you very much Nagyon szépen köszönöm

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