Why is mathematics beautiful?

László Lovász

Loránd Eötvös University

and

Alfréd Rényi Institute of Mathematics

Or is it?

It is not the most beloved subject in schools…

To appreciate mathematics

→ Beauty, internal harmony, intellectual challange

→ Usefullness, understanding phenomena

Quote from a mathematician

The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics.

G.H. Hardy

Quote from a logician

Mathematics, rightly viewed, possesses not only truth, but supreme beauty — a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show.

Bertrand Russel

Quotes from a physicists

To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature ...

Richard Feynman

Elegance should be left to shoemakers and tailors.

Ludwig Boltzmann

Quotes from “everyday life”

Tao and co-author Allen Knutson produced beautiful work on a problem known as Horn's conjecture…

Report on the work of Fields medalist Terence Tao

Zermelo gave a beautiful proof that every set can be well ordered…

Daniel Grayson lecture notes on the internet

Hey guys … $\operatorname{Arctan}(x)+\operatorname{Arctan}(\frac{1}{x})=\frac{\pi}{2}$ …

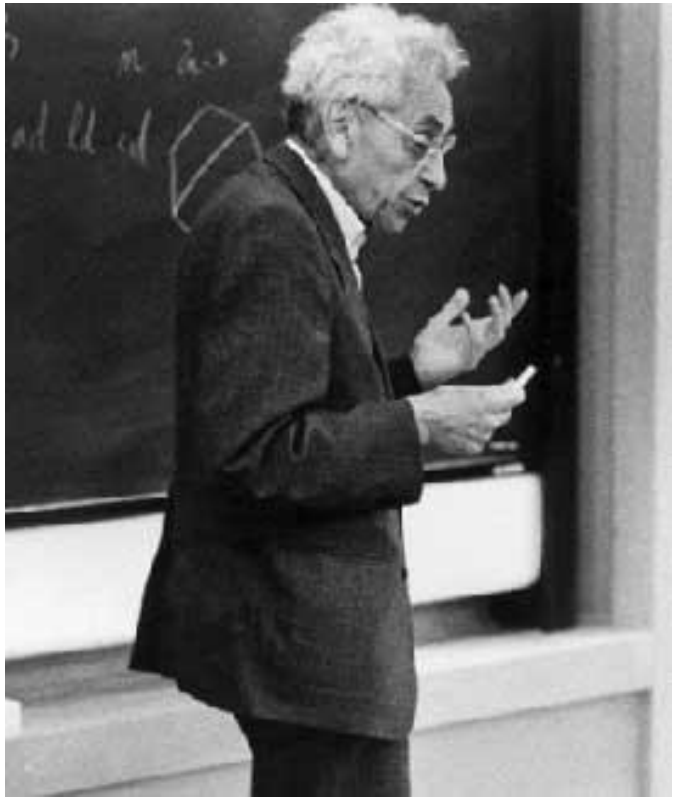
Just wondering what is the most elegant proof of this?

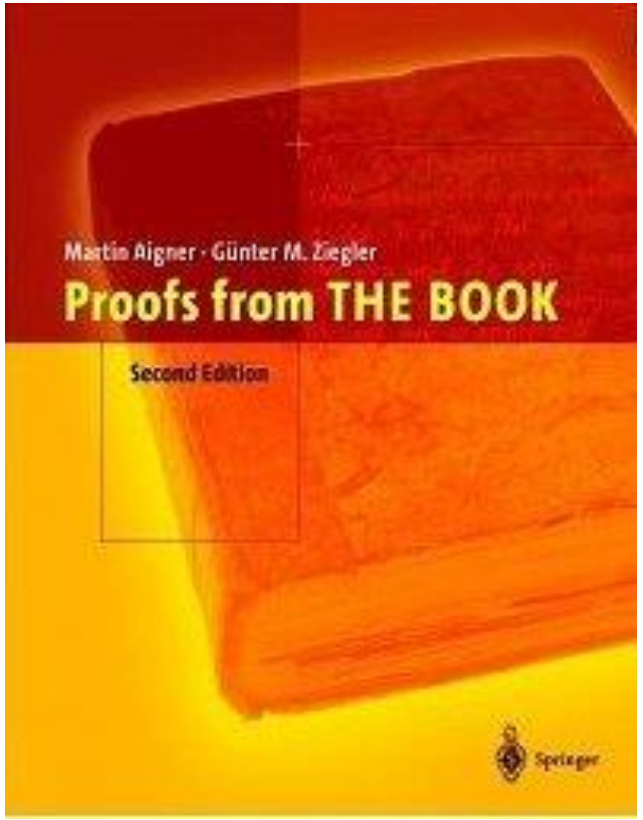
From an internet forum

The Book with the most beautiful proof of every theorem

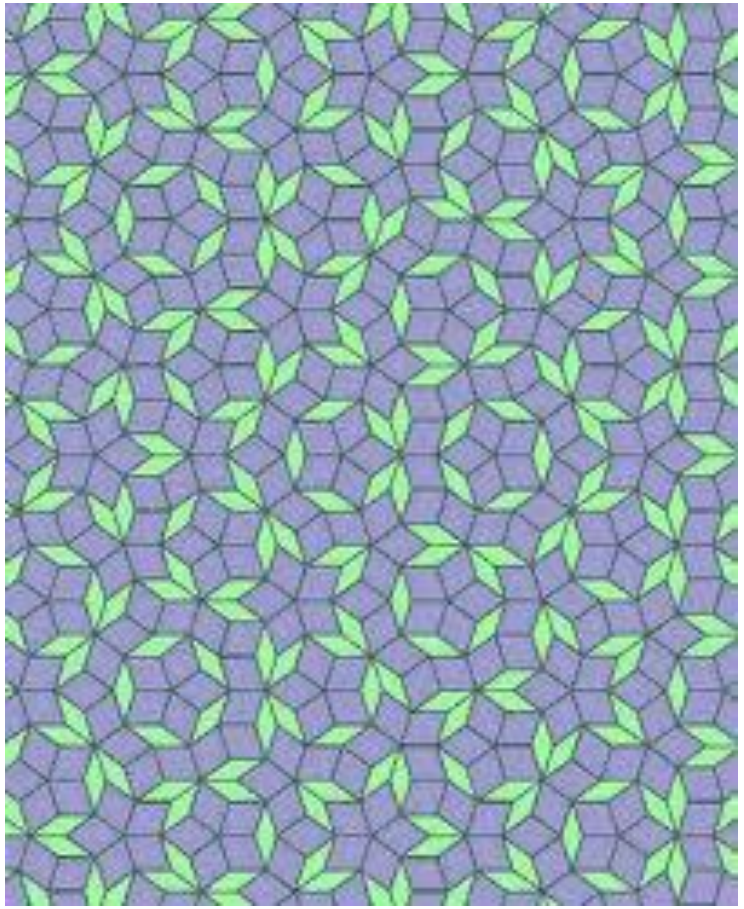
„This is straight from The Book.”

Paul Erdős

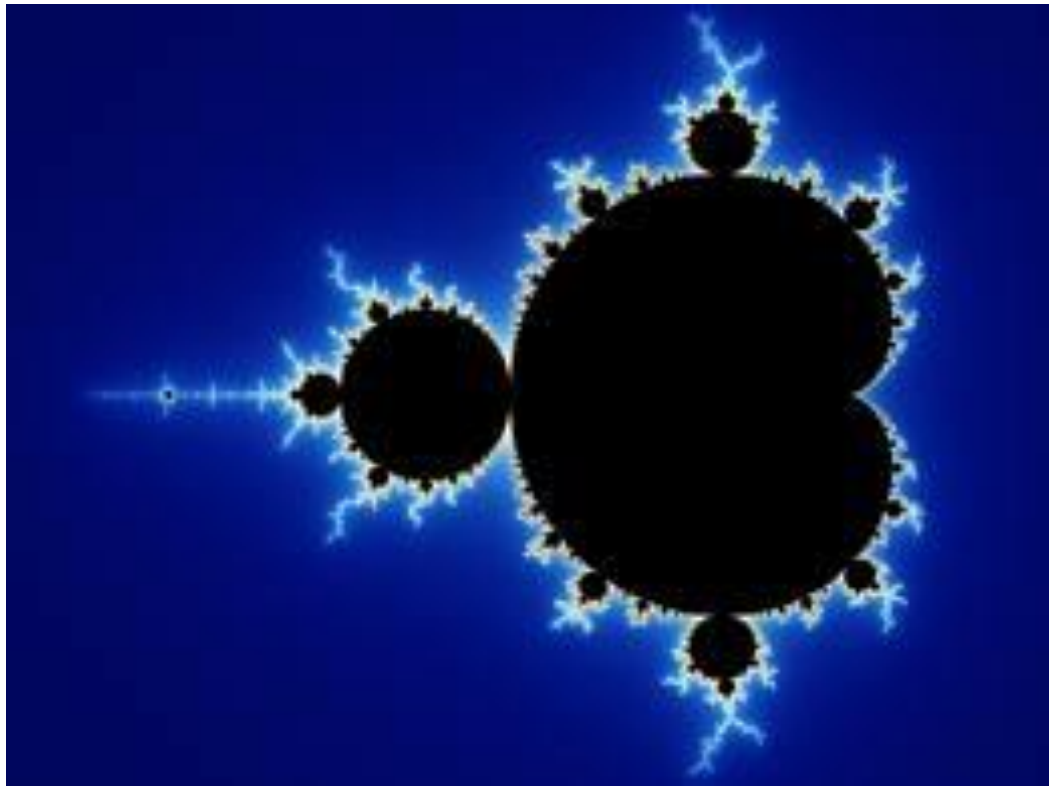


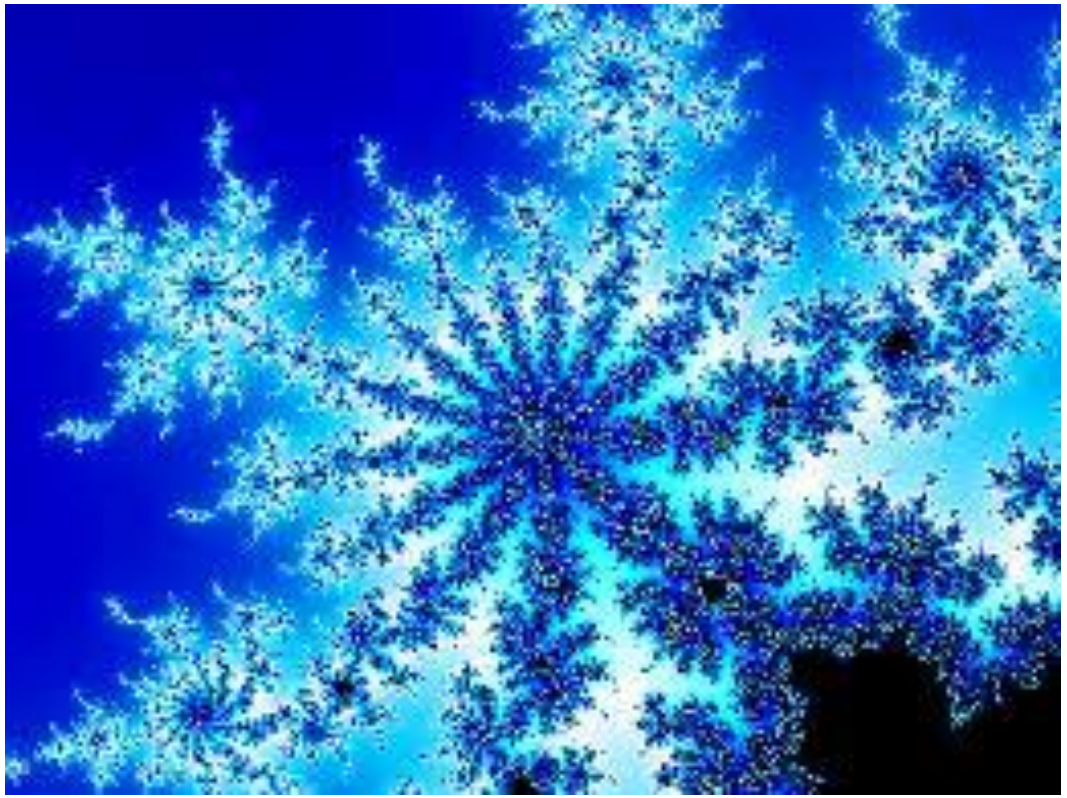


Beautiful objects: tilings



Beautiful objects: fractals





A beautiful formula

Euler's Identity:

$$e^{i\pi} = -1$$

$\pi = 3.141592653589793\ldots$

$i = \sqrt{-1}$

$e = 2.718281828459045\ldots$

geometry

algebra

analysis

The other side: applications

Reprinted from *Communications in Pure and Applied Mathematics*, Vol. 13, No. I (February 1960). New York: John Wiley & Sons, Inc.

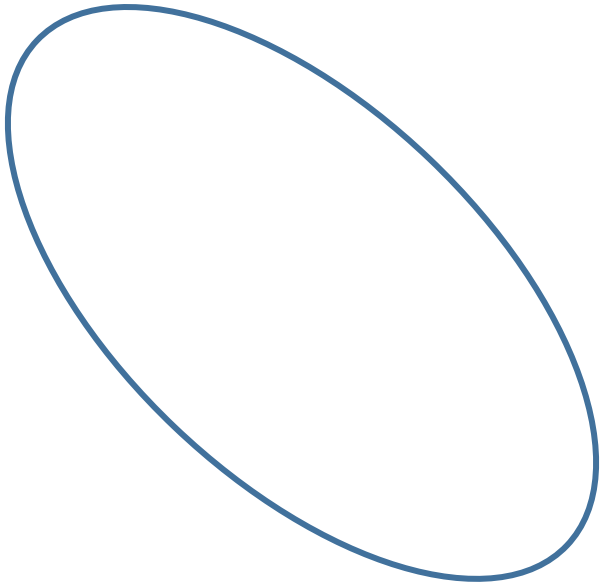
THE UNREASONABLE EFFECTIVENSS OF MATHEMATICS IN THE NATURAL SCIENCES

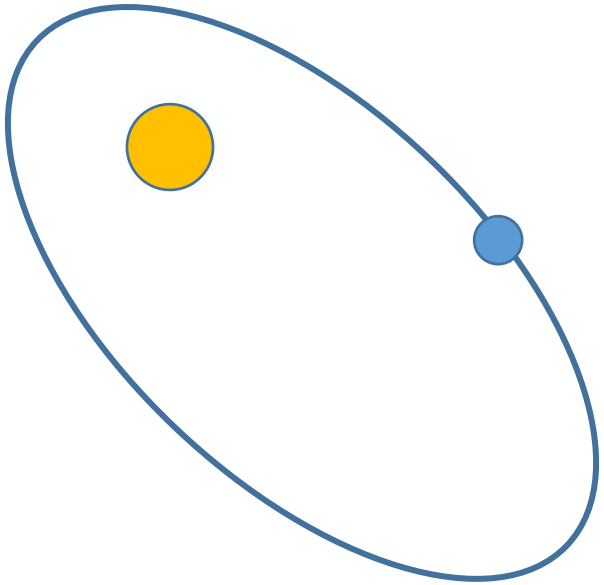
Eugene Wigner

"The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve."

Applications: a “killer app”

Ellipses (Apollonios) >>> orbits of planets (Kepler)





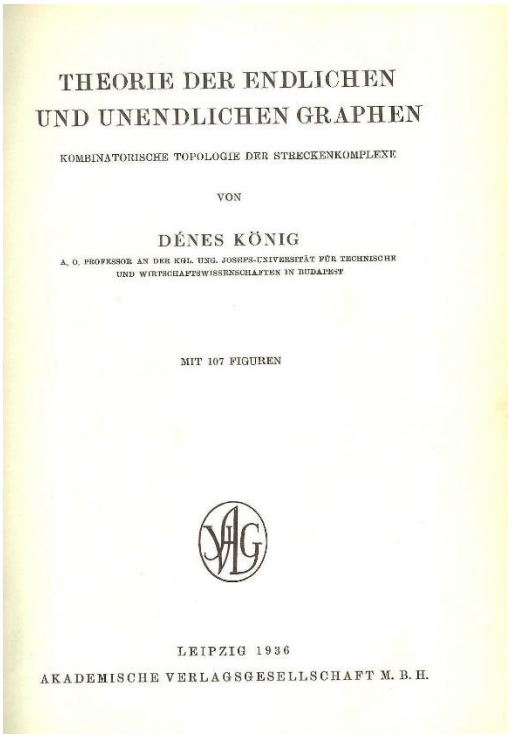
Graph theory: a more personal view

Königsberg bridges *1736*

Leonard Euler

Kőnig's book *1936*

Dénes Kőnig



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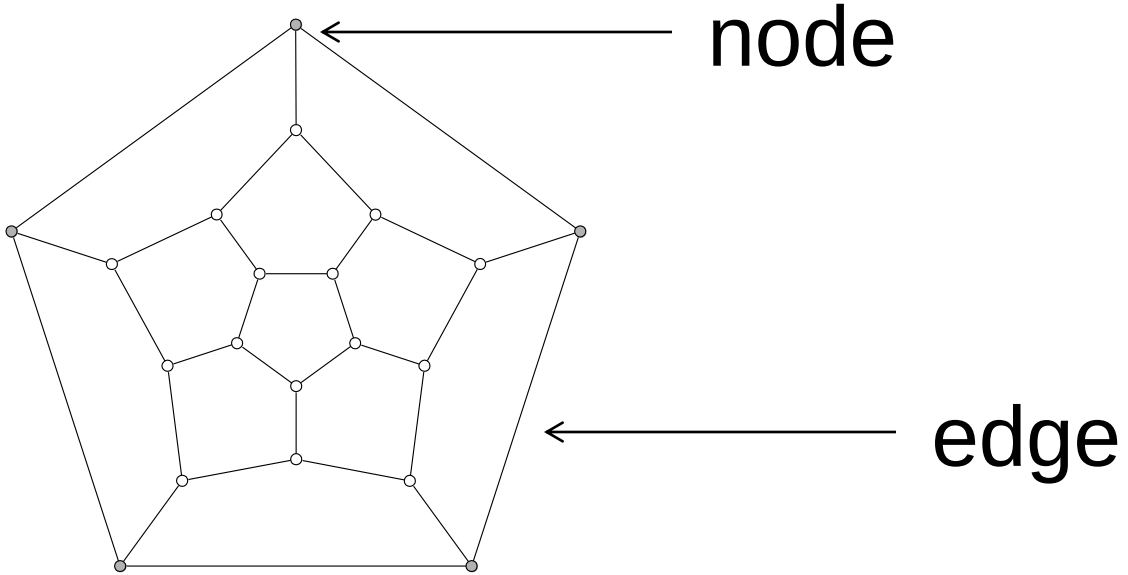
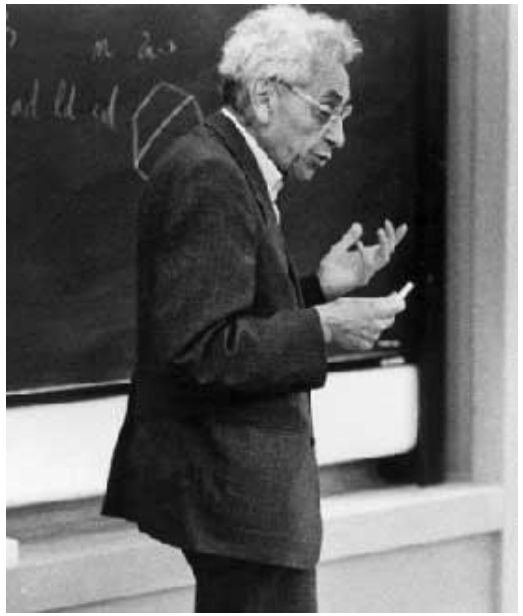
Graph and networks



Simple graph problems to start



Paul Erdős

Lajos Pósa

The edges of any graph with n nodes can be covered by $n^2/4$ edges and triangles.

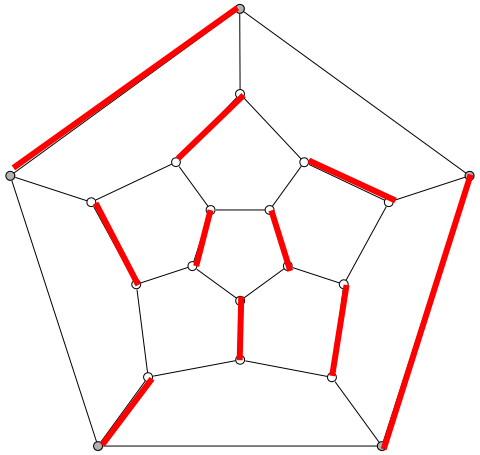
Erdős-Goodman-Pósa

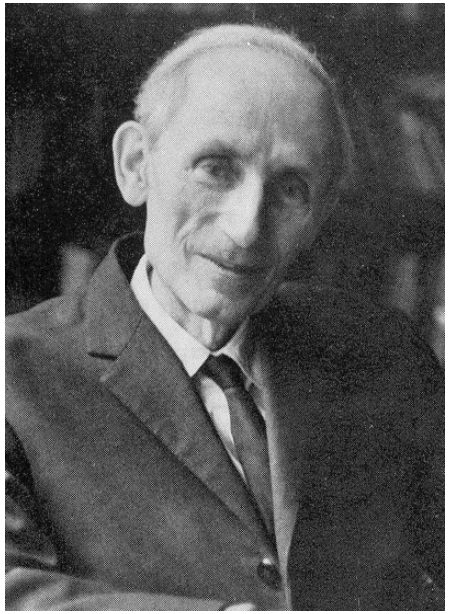
Graph theory meets computer science

Perfect matching problem:

Can the nodes of a graph

be paired up by edges?





Tibor Gallai

Graph theory meets computer science

Perfect Matching Problem

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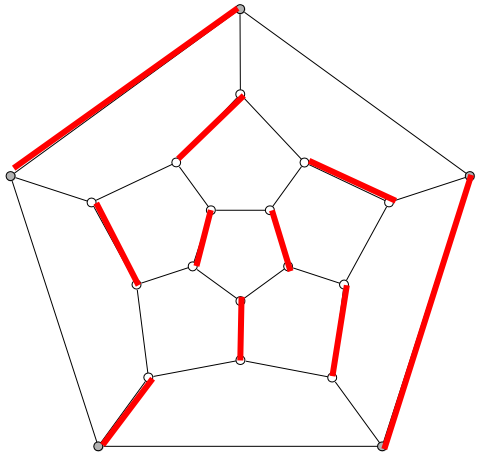
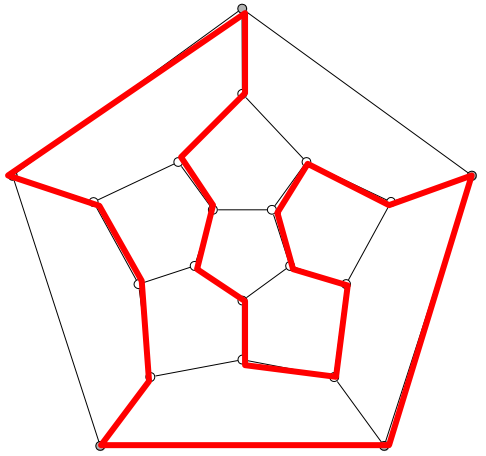


Traveling Salesman Problem

Can all nodes be traversed

exactly once (Hamilton cycle)?



Graph theory meets computer science

Perfect M… …blem

C… …aph

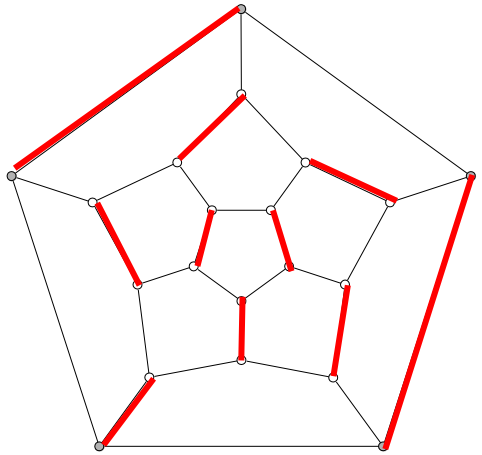
Travel… …em

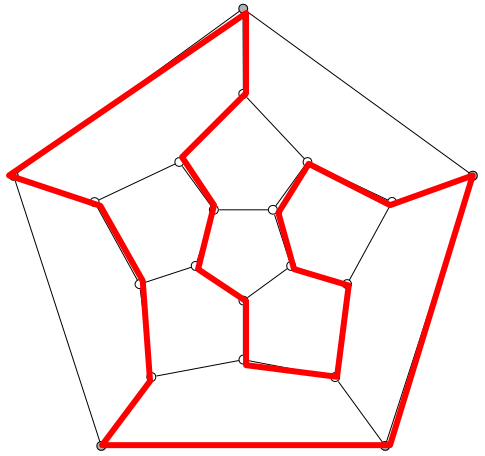
…aversed

…e (Hamilton cycle)?

"Easy": efficient algorithm, nice structure

Hard: no efficient algorithm, no nice structure





Graph theory meets computer science

Perfect M… …blem

C… …aph

Travel… *…ge (Hamilton cycle)?*

"Easy": efficient algorithm, nice structure

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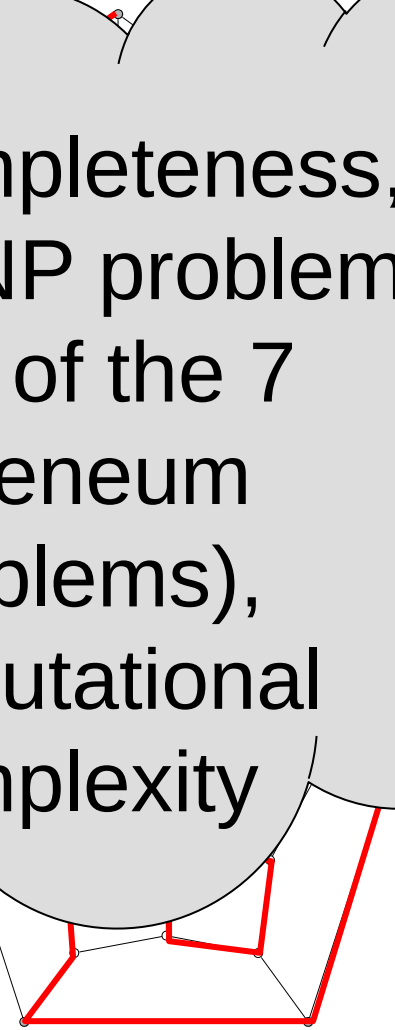
NP-completeness, the P=NP problem (one of the 7 Milleneum Problems), computational complexity



Very large graphs and networks

internet

social networks

ecological networks

computer chips

crystals

brain

universe

"The laws of nature are written
in the language of differential equations."

New mathematical language of science.
What can math say about it?

What properties to study?

How to obtain information about them?

How to model them?

How to approximate them?

Covid 19 pandemic

How to get information about the network of all people?

- local information: contact search
- global information: daily pandemic data
  number of people commuting
  masks

Graph limit theory, Microsoft Research

The infinite is an approximation of the very large finite.

Christian Borgs,
Jennifer Chayes

Vera T. Sós

Balázs Szegedy

Katalin Vesztergombi,
LL

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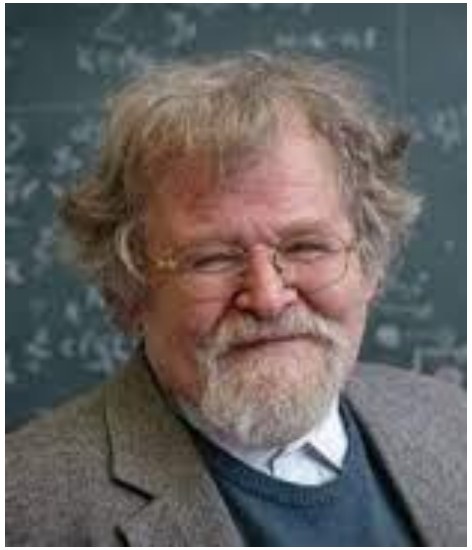
Graph limit theory, ELTE, Rényi Institute and ERC

Albert-László Barabási



Jarik Nesetril

Miklós Abért

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How could (should?) trends in mathematics research change education?

The size of the community and of mathematical research activity is increasing very fast.

New areas of application, and their increasing significance.

New forms of mathematical activity

New tools: computers and information technology.

The size of math

The number of mathematical publications *increases exponentially*. One person can only be familiar with a small corner of mathematics. More and more coauthors of papers.

In larger and more complex societies an increasingly large fraction of resources is devoted to *non-productive activities*: Colloquium talks, conferences, workshops, summer schools, …

Secondary processing of research results becomes more and more important: expository writing, popularization, Wikipedia, blogs,…

New areas of application

Traditional areas of application: physics and engineering.
Main math tool: analysis

New areas of application: computer science, biology, operations research, neuroscience, social sciences, …
Math tools: probability, discrete mathematics

Which branches of mathematics will be applicable in the near future?
Unpredictable.

Pacifism and the usefulness of mathematics

„No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world.”

Godfrey Harald Hardy

Prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,…

>>> public key cryptography, RSA code 1978

New forms of mathematical activity

Paul Erdős: the Art of Conjecturing

Blogs by leading mathematicians

Polymath Project (started by Timothy Gowers in 2009): Solved open problem about the density version of the Hales–Jewett theorem.

Highschool version: CrowdMath Project by MIT

Developments in math and the math curriculum

What to teach in secondary school:

- classical (elementary geometry, algebra, number theory, calculus)

- newer (set theory, probability, discrete math)

- mathematical competencies (problem solving, abstraction, generalization and specialization, logical reasoning, use of mathematical formalism

- communicate math: read and write up; read and tell to a friend; read and give a presentation

This is an overwhelming amount of goals!

New tools: computers and information technology should help.

Heretical thought: expository style teaching?

Thank you for your attention!